Brantina Chirinda[1]
Graduate School of Education
University of California, Berkeley
United States

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FROM PROCEDURES TO POSSIBILITIES: REFLECTIVE TEACHER ADAPTATION TO MATHEMATICAL PROBLEM SOLVING WITH FERMI PROBLEMS

Abstract: Fermi problems are open-ended estimation tasks that require flexible reasoning, assumption-making, and approximation. While research highlights their potential to foster learner engagement, less is known about teacher adaptation to these open-ended approaches, especially in contexts of disadvantage. This study investigated the professional learning of a mathematics teacher as he implemented Fermi problems in a disadvantaged South African classroom. An instrumental qualitative case study was used, drawing on classroom observations, semi-structured interviews, and reflective field notes collected over six weeks. Analysis was guided by Schon's concept of the reflective practitioner and Clarke and Hollingsworth's Interconnected Model of Professional Growth. Findings show how the teacher's practice evolved from procedural delivery toward adaptive facilitation that valued uncertainty, multiple strategies, and learner autonomy. Through iterative reflection, the teacher developed greater confidence, responsiveness, and pedagogical insight into problem-solving instruction. The findings inform teacher education by illustrating how reflective engagement with open-ended tasks can cultivate adaptive expertise within everyday practice. The study contributes to mathematics education by demonstrating how Fermi problems can function as a catalyst for teacher professional learning, especially in settings where formal professional development opportunities are limited.

Keywords: Fermi problems; teacher professional learning; reflective practice; teacher reflection; mathematical problem solving.

Introduction

Open-ended problem solving remains a persistent challenge in mathematics classrooms worldwide, particularly in contexts where teachers are accustomed to procedural instruction, and learners expect single, closed-form answers. Mathematics education reforms across many countries, including South Africa's Curriculum and Assessment Policy Statement (CAPS), emphasize reasoning, problem solving, and the application of knowledge to unfamiliar contexts as key goals for developing 21st-century competencies (South African Department of Basic Education [DBE], 2011; Schoenfeld, 2016). Despite these curricular aspirations, classroom practice in many schools located in contexts of disadvantage—geographical locations where people with low or no income reside—continues to be dominated by rote-based, procedural teaching (Chirinda & Barmby, 2018). Teachers working in contexts of disadvantage face multiple constraints—overcrowded classes, limited materials, and linguistic diversity—that make it difficult to implement learner-centered pedagogies (Chirinda & Barmby, 2017; Spaull, 2015).

One promising approach to bridging this gap is the use of Fermi problems—open-ended estimation tasks that emphasize structured reasoning and invite learners to explore multiple solution pathways to real-world situations (Segura & Ferrando, 2023). Crucially, their low cost, open-ended nature, and

[1] brantinac@berkeley.edu ORCID ID:https://orcid.org/0000-0003-3142-7149

reliance on learners' lived experiences make them accessible and adaptable pedagogical tools for promoting reasoning in resource-constrained environments.

However, facilitating Fermi problems presents distinctive challenges for teachers. Unlike traditional exercises, these tasks require teachers to manage uncertainty, scaffold assumptions, and orchestrate discussions that validate multiple reasonable answers. Teachers accustomed to procedural instruction often struggle with the open-endedness of such problems, feeling discomfort when learners produce divergent solutions or when classroom discussions become noisy and unpredictable. These demands are intensified in large, multilingual, and under-resourced classrooms, where facilitating dialogue and reasoning can be particularly difficult. As a result, implementing Fermi problems is not only a matter of task design but also a significant problem of teacher learning.

Research on professional development (PD) suggests that the most meaningful teacher learning occurs in situ—through cycles of experimentation, reflection, and adaptation within authentic classroom contexts (Clarke & Hollingsworth, 2002; Schon, 1983). In this view, the classroom itself becomes a site of professional growth, as teachers reflect on their facilitation moves, respond to learners' reasoning, and refine their instructional strategies over time. Within this framework, facilitating Fermi problems may serve as a form of situated PD, providing teachers with opportunities to examine and adjust their pedagogical assumptions in real time.

Despite growing international interest in Fermi problems and mathematical modeling, little research has explored how teachers adapt to facilitating these open-ended tasks, particularly in disadvantaged or resource-constrained environments. Understanding this process is crucial, as teachers' confidence, beliefs, and facilitation skills shape whether classroom diversity of thought becomes productive discourse or unproductive confusion (Schoenfeld et al., 2023).

This study addresses that gap by examining the professional learning of a secondary mathematics teacher in a disadvantaged South African classroom as he implemented Fermi problems over 6 weeks. Drawing on Schon's (1983) concept of the *reflective practitioner* and Clarke and Hollingsworth's (2002) *interconnected model of professional growth (IMPG)*, the study investigates how the teacher's beliefs, confidence, and instructional strategies evolved through cycles of reflection-in-action and reflection-on-action. By foregrounding the teacher's perspective, the study illuminates how engaging with open-ended problem solving can foster professional adaptation and pedagogical transformation in challenging educational settings. It contributes to the growing body of literature demonstrating that meaningful teacher learning can occur not only through formal workshops but through reflective engagement with innovative classroom practice. This study extends pedagogical research by conceptualizing classroom-based reflection as situated PD in under-resourced schools. This study was guided by the following research questions:

1. How does a mathematics teacher experience the process of facilitating Fermi problems in a disadvantaged secondary classroom?
2. What challenges and opportunities arise as the teacher shifts from procedural to open-ended, inquiry-oriented pedagogy?
3. How do the teacher's beliefs, confidence, and facilitation strategies evolve through iterative reflection and practice over time?

Literature review

The preceding discussion highlights the complex demands teachers face when facilitating open-ended mathematical tasks such as Fermi problems, particularly in classrooms located in contexts of disadvantage. Existing research on open-ended mathematical problem solving, teacher facilitation, and professional learning provides insight into how teachers navigate and learn from these challenges.

Open-Ended Mathematical Tasks and Classroom Interactions

Research on open-ended mathematical tasks has grown steadily in recent years, emphasizing their potential to transform both learner reasoning and classroom dynamics. Classroom studies show that such tasks generate a diversity of solution pathways and emphasize iterative refinement, approximation, and social negotiation among learners (Ferrando & Albarracin, 2021). These characteristics position open-ended tasks as powerful tools for developing higher-order thinking but also as pedagogical challenges that require significant facilitation skills from teachers (Segura & Ferrando, 2023). Because Fermi problems invite multiple reasonable solutions and draw on learners' everyday experiences, they are particularly suitable for resource-constrained classrooms. They are low-cost, adaptable, and capable of engaging diverse learners in meaningful mathematical reasoning and estimation (Ärlebäck, 2009).

Teacher Facilitation in Open-Ended Contexts

The shift from closed to open-ended problem-solving tasks fundamentally alters the teacher's instructional role. In such contexts, the teacher becomes an orchestration agent, responsible for guiding inquiry, managing uncertainty, and sustaining mathematical dialogue. Studies mapping teacher "moves" during modeling and open-ended tasks highlight practices such as revoicing learner ideas, posing probing comparative questions, and prompting justification of assumptions as key forms of support that help learners navigate the modeling process (Albarracin & Ärlebäck, 2025). Effective facilitation is thus not incidental—it shapes whether classroom divergence becomes productive mathematical discourse or simply noise (Ferrando & Albarracin, 2021).

For teachers accustomed to procedural instruction, these facilitation demands can be daunting. Research on teachers implementing modeling tasks shows frequent discomfort with ambiguity, difficulties managing divergent solutions, and uncertainty about when and how to intervene (Segura & Ferrando, 2023). These challenges are magnified in under-resourced classrooms, where diverse proficiency levels and limited materials constrain opportunities for extended discussion (Chirinda & Barmby, 2017; Spaull, 2015).

Teacher Knowledge, Beliefs, and Attitudes Toward Modeling

A parallel body of research examines teachers' beliefs, confidence, and pedagogical knowledge regarding modeling and open-ended problem solving. Surveys and mixed-methods studies show that many in-service teachers hold positive attitudes toward modeling but report limited familiarity with the facilitation moves required to support it effectively (Blum & Leiss, 2007).

The complexity of facilitating open-ended tasks means that teacher learning cannot be reduced to procedural knowledge. Instead, it involves epistemological shifts—teachers must reconceptualize mathematics as an exploratory, reasoning-driven activity rather than as a collection of fixed procedures. Studies indicate that such shifts occur gradually, often through experiential engagement with open-ended tasks and opportunities to reflect on learners' reasoning (Clarke & Hollingsworth, 2002; Schon, 1983).

Professional Development and Situated Teacher Learning

Given these pedagogical demands, research on teacher PD and situated learning has become increasingly relevant. Meta-analyses and program evaluations consistently show that PD yields stronger outcomes when it incorporates sustained, practice-based elements—such as video analysis, lesson study, or coached enactment—rather than brief workshops (Desimone, 2009). The most impactful PD allows teachers to experiment, reflect, and adapt within their own classrooms, supporting iterative cycles of improvement.

Recent studies demonstrate that classroom-based reflection fosters meaningful change in teachers' instructional practice (Korthagen, 2017). Through reflection-in-action and reflection-on-action, teachers learn to notice learner thinking, adjust facilitation strategies in real time, and revise assumptions about what counts as productive mathematical activity (Schon, 1983).

Reflective Practice and Teacher Growth

Reflective practice theory provides a robust lens for understanding how teachers develop adaptive expertise (Schon, 1983; Korthagen, 2017). In mathematics education, reflective practice has been linked to teachers' ability to notice learners' reasoning, respond flexibly, and evolve from directive to facilitative roles (Schoenfeld, 2016). Empirical work shows that reflective tools—such as guided prompts, video-based reflection, and peer dialogue—enhance teachers' ability to negotiate the uncertainties of open-ended classroom tasks (Ferrando & Albarracin, 2021). These insights suggest that Fermi problems are more than a pedagogical tool for learners—serving as mechanisms for teachers 'reflective adaptation and professional growth.

Theoretical Framework

This study draws on two complementary theoretical perspectives—Schon's (1983) Reflective Practitioner and Clarke and Hollingsworth's (2002) IMPG—to examine how a teacher's professional knowledge, beliefs, and practices evolved through the enactment of Fermi problems. Together, these frameworks conceptualize professional growth as an iterative process of reflection and change, grounded in practice and responsive to contextual demands.

Schon's Reflective Practitioner

Schon (1983) proposed that professionals develop expertise through reflection that occurs both in and on action. Reflection-in-action refers to the practitioner's ability to think on their feet—to notice, interpret, and respond to emerging situations in real time. Reflection-on-action, by contrast, takes place after the event, when practitioners review their experiences to derive insights that inform future actions. In teaching, these two modes of reflection are interdependent: as teachers experiment with pedagogical moves, they adapt responsively to classroom dynamics and subsequently reconstruct their professional understanding.

In mathematics education, reflective practice enables teachers to notice learners' reasoning, interrogate their own assumptions, and refine their facilitation strategies (Mason, 2011). When teachers encounter open-ended problems such as Fermi tasks—where no single correct answer exists—they are compelled to navigate ambiguity, interpret diverse learner responses, and decide which contributions to extend or challenge. These moments of uncertainty activate Schon's reflective cycle: teachers frame the situation, act experimentally, and reframe based on outcomes. Over time, repeated cycles of reflection foster adaptive expertise (Hatano & Inagaki, 1986)—the capacity to flexibly apply professional knowledge in novel contexts rather than reproduce routine procedures.

Within disadvantaged classrooms, reflective practice becomes a crucial tool for professional learning. Teachers cannot rely solely on pre-planned procedures; they must make real-time pedagogical judgments, negotiate meaning across languages, and orchestrate collective reasoning among learners with differing abilities. Schon's (1983) framework thus illuminates how facilitating Fermi problems can function as a practice-based form of professional learning, prompting teachers to experiment, interpret, and refine their facilitation in response to evolving classroom realities.

Clarke and Hollingsworth's Interconnected Model of Professional Growth

Clarke and Hollingsworth (2002) reconceptualized teacher professional growth as a process of change across four interrelated domains:

1. **The Personal Domain** (teacher knowledge, beliefs, and attitudes);
2. **The Domain of Practice** (professional experimentation and classroom action);
3. **The Domain of Consequence** (salient outcomes perceived from practice, such as learner engagement or understanding); and
4. **The External Domain** (sources of information or stimulus, including curriculum reforms, research input, or collaboration).

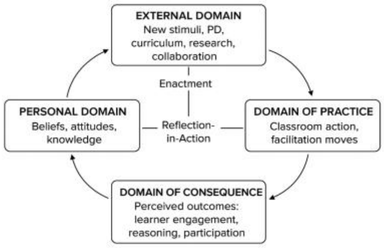
Growth occurs through enactment (putting ideas into practice) and reflection (drawing meaning from experience). These two processes connect the domains in multiple pathways—forming a change network rather than a linear sequence. The model's strength lies in recognizing that teacher learning is non-linear, recursive, and context-sensitive: new insights in one domain may trigger shifts in another, depending on how teachers interpret and respond to their experiences.

In this study, the IMPG offered a structural lens for tracing how engagement with Fermi problems stimulated professional growth. The External Domain comprised the introduction of Fermi problems and researcher support. The Domain of Practice involved the teacher's enactment of these problems in an overcrowded Grade 9 classroom. The Domain of Consequence included both perceived learner engagement and the teacher's recognition of changes in classroom discourse. Reflection on these outcomes led to changes in the Personal Domain—for instance, shifts in beliefs about control, correctness, and the value of exploratory reasoning. Through repeated cycles of reflection and enactment, the teacher's practice evolved from procedural delivery toward dialogic, inquiry-oriented facilitation.

Integrating the Two Frameworks

While Schon's (1983) framework emphasizes how professional learning unfolds through reflection, Clarke and Hollingsworth's (2002) model maps where change occurs within the professional system. Integrating these perspectives provides both the process and structure needed to understand teacher growth during the Fermi problem intervention. Reflection-in-action represents the mechanism driving change across domains, while the IMPG captures the trajectory of these changes over time. This integrated framework (Figure 1) thus positions the implementation of Fermi problems as a situated PD process rather than merely an instructional intervention. It highlights how teaching open-ended, uncertain tasks can itself become a context for professional learning—particularly in under-resourced schools where formal development opportunities are scarce.

Together, these frameworks offer a robust analytical tool for understanding how teachers learn,



adapt, and transform their practice through everyday classroom challenges.

Figure 1. *Integration of Schon's (1983) Reflective Practitioner and Clarke & Hollingsworth's (2002)* Interconnected Model of Professional Growth.

Figure 1 visually demonstrates that reflection-in-action operates as the mechanism linking the Personal and Practice domains during teaching. At the same time, reflection-on-action connects outcomes (Domain of Consequence) back to belief and knowledge systems (Personal Domain). Enactment processes connect the External Domain (new pedagogical stimuli) to classroom practice, creating an iterative, cyclical system of professional learning.

Methodology

Research Design

Employing an instrumental qualitative case study (Yin, 2018), this study provides an in-depth, situated understanding of a teacher's reflective adaptation over a six-week intervention. It details the evolution of the teacher's pedagogical actions and beliefs while facilitating open-ended mathematical tasks in a disadvantaged classroom. Teacher learning was conceptualized using Schön's (1983) reflective practitioner and the IMPG (Clarke & Hollingsworth, 2002) as iterative reflection across personal, practice, and consequence domains.

Research Context and Participant

The study was conducted at Mahlathi Secondary School (pseudonym), a low-fee public school located in a peri-urban township on the outskirts of Gauteng Province, South Africa, a context facing significant structural and pedagogical challenges in mathematics education (Spaull, 2015). The school serves approximately 620 learners from surrounding informal settlements and low-income housing areas. Learners typically speak isiZulu as their home language, with English used as the medium of instruction from Grade 4 onwards. Class sizes average 50–60 learners, and access to technology and manipulatives is limited. The mathematics department consists of four teachers, who meet weekly for informal lesson planning. The broader school culture is one of commitment and community support, despite material constraints.

The focal participant, Mr Langa (pseudonym), an experienced Grade 9 mathematics teacher with 12 years of teaching experience at Mahlathi Secondary, was purposively selected due to his willingness to engage in the non-routine pedagogical demands of the intervention. He held a Bachelor's degree in Mathematics Education and was familiar with the South African CAPS. However, he had limited prior exposure to inquiry-based pedagogies. During the 6-week intervention, he implemented one Fermi problem per week with his Grade 9 class and maintained a reflective journal after each lesson. The focus on a single teacher enabled intensive engagement over time and supported detailed documentation of reflective processes and pedagogical adaptation.

The Fermi Problem Intervention

A one-day training introduced the teacher to the principles of Fermi problem facilitation. The session introduced him to the Teacher Facilitation Guide for Fermi Problems (Appendix A), which served as a practical resource outlining facilitation strategies, reflection prompts, and approaches to promote learner engagement during the intervention.

Over six weeks, Mr Langa implemented a sequence of estimation challenges that drew on learners' immediate environment, including: estimating the number of windows in the school, the number of plastic bottles sold weekly at the tuckshop, estimating the number of learners who could fit into the school hall, the number of bottles needed to fill a pool, the total steps from the school gate to the staffroom, and the number of chalk pieces used in a term. Each task was open-ended, required assumptions, and invited multiple solution paths. The problems were collaboratively designed with the researcher to ensure alignment with Grade 9 mathematics topics and local relevance.

Lessons followed a consistent cycle: the teacher introduced the problem, groups discussed assumptions and solution strategies, and learners presented and compared estimates during whole-class discussion. Sessions concluded with a joint reflection on how assumptions and estimation informed reasoning. These discussions promoted collaboration and reasoning while positioning estimation as a legitimate mathematical process. For this study, the Fermi sessions functioned as the professional learning site for Mr. Langa. Each activity provided opportunities for reflection-in-action during facilitation and reflection-on-action afterward.

Data Collection

Four complementary forms of data were gathered to capture both the teacher's evolving facilitation practices and his reflective learning processes.

Classroom Observations: Classroom observations were conducted for all six Fermi-problem lessons using a structured observation guide that captured the teacher's questioning patterns, facilitation strategies, and responsiveness to learner ideas. Field notes documented critical incidents where the teacher's decisions or reflections signaled professional growth or tension.

Teacher Reflective Journals: After each session, Mr. Langa completed a brief reflective journal prompted by questions such as: *What surprised you today?*" and *How did learners' responses influence your teaching decisions?*" These journals captured the teacher's evolving interpretations and beliefs about his practice.

Semi-Structured Interviews: Interviews focused on the teacher's reflections, perceived learner responses, facilitation challenges, and insights into his professional growth. The interview protocol aligned with the IMPG domains, probing changes in beliefs, practices, and perceived outcomes. Each interview lasted approximately 20–30 minutes, was audio-recorded, and transcribed verbatim for analysis.

Researcher reflective notes and analytic memos were written after each session to document contextual observations, interactions, emerging interpretations, and reflexive insights about teacher learning. These notes provided a secondary layer of reflection and supported the data triangulation.

Data Analysis

Thematic analysis followed Braun and Clarke's (2006) six-phase framework. Deductive codes were informed by Schon's (1983) reflective practitioner model and Clarke and Hollingsworth's (2002) IMPG, while inductive codes emerged directly from the data. The process involved five key steps:

1. **Familiarization:** Reading and rereading transcripts and field notes to identify recurring patterns of reflection and change.
2. **Initial Coding:** Generating codes such as reframing classroom noise," "valuing learner ideas," "tension between control and openness," and emergent confidence."
3. **Categorization:** Grouping codes into broader categories representing dimensions of professional growth (e.g., adaptive facilitation," "evolving beliefs about mathematics," "pedagogical risk-taking").
4. **Mapping to Theoretical Domains:** Codes were organized within Clarke and Hollingsworth's (2002) four domains, identifying how reflection connected them through enactment or feedback loops.
5. **Interpretive Synthesis:** Schon's (1983) distinction between reflection-in-action and reflection-on-action was used to interpret how the teacher's insights emerged from classroom experimentation.

This dual-level analysis enabled both descriptive representation of change and interpretive explanation of how reflective processes drove professional growth. Table 1 shows the progression from raw data to themes using thematic analysis (Braun & Clarke, 2006) and drawing on Schon's reflective practitioner theory and Clarke & Hollingsworth's model of professional growth as analytical lenses.

Table 1: *Summary of Data Analysis Process*

Data Source	Initial Coding Focus	Analytical Procedures	Example Codes	Resulting Themes
Semi-structured interviews	Teacher beliefs, perceptions, and reflections on Fermi problem facilitation	Transcripts read repeatedly for familiarization; deductive and inductive coding applied; iterative comparison across three interview phases	*Uncertainty with open tasks, learner autonomy, teacher confidence, shift in beliefs*	1. Navigating uncertainty 2. Evolving teacher confidence 3. Reframing mathematical learning
Classroom observations	Teacher facilitation moves, questioning patterns, and learner interactions	Observation notes and video transcripts coded for visible facilitation strategies; identification of reflection-in-action moments	*Revoicing, scaffolding assumptions, managing noise, prompting justification*	4. Adaptive facilitation practices 5. Orchestrating productive dialogue

Researcher reflective notes	Researcher interpretations, contextual insights, and teacher reflection-on-action	Memos analyzed to trace teacher growth across lessons; triangulated with interviews and observations	*Shifting focus from control to exploration, noticing learner reasoning, self-questioning*	6. Reflective adaptation and professional growth

Trustworthiness

Trustworthiness was ensured through triangulation, member checking, thick description, and reflexive journaling (Lincoln & Guba, 1985). Triangulation was achieved by comparing insights from observations, interviews, and journals. Member checking involved sharing preliminary interpretations with Mr. Langa to confirm accuracy. Thick contextual description enhanced transferability to other resource-constrained environments. Reflexivity was maintained through the researcher's analytic memos to minimize bias.

Ethical Considerations

Ethical approval was obtained from the Gauteng Department of Education, the participating school, and the host university's ethics committee. The teacher provided informed consent, and a pseudonym, Mr. Langa, was used to protect his identity. Participation was voluntary, and he could withdraw at any time during the study without negative consequences.

Results

The findings reported here emerged within a research–practice partnership; the researcher's dual role as facilitator and observer likely shaped the reflective trajectory described, and the professional growth documented should be understood as co-constructed rather than as entirely independent change. This section presents the findings of the 6-week classroom-based intervention in which the teacher, Mr Langa, implemented Fermi problems as part of his Grade 9 mathematics instruction. The analysis traced the trajectory of professional growth, illustrating how his facilitation practices, beliefs, and confidence evolved through iterative cycles of reflection-in-action and reflection-on-action.

The results are organized into three major themes that emerged from the data:

1. Initial Uncertainty and the Challenge of Letting Go of Control
2. Emergent Reflection and Adaptive Facilitation
3. Consolidated Pedagogical Insight and Professional Confidence

Each theme integrates evidence from classroom observations, post-lesson interviews, and teacher reflective journals.

Initial Uncertainty and the Challenge of Letting Go of Control

At the start of the intervention, Mr Langa expressed both enthusiasm and apprehension about using open-ended problems. In the first interview, he remarked,
"*I am used to lessons where I show the method and they follow. Now, everyone has a different answer—I feel like I'm losing control.*"

This discomfort reflected his deeply held belief that good teaching involves ensuring learners reach correct answers quickly and efficiently. Early observations confirmed this tension. During the first Fermi task in week 1 - "*How many people can fit in the school hall?"*—he frequently stepped in to correct learners or standardize their assumptions.

Journal, Week 1 - Grappling with loss of control
Mr Langa displayed initial uncertainty: "*I wanted to guide learners to the right answer, but then I realized there wasn't just one answer. I caught myself giving hints too soon. It felt uncomfortable not to control everything."*

Mr Langa's early reflection showed discomfort with learner autonomy and a preference for procedural guidance, consistent with his prior teaching style.

Observation notes described his movement between groups as *directive and evaluative*, often rephrasing learner statements into conventional mathematical terminology. According to Clarke and Hollingsworth's IMPG framework, these moments signified initial activation of the external domain (exposure to the Fermi problem approach) and first enactment in the domain of practice. Schon's (1983) notion of reflection-in-action was evident as Mr Langa paused mid-lesson, questioning his instinct to correct learners. Although he reverted to procedural habits, his post-lesson reflection signaled growing awareness of the tension between control and openness - a necessary precondition for professional growth.

Emergent Reflection and Adaptive Facilitation

Mr Langa's journal entries indicated that he gradually shifted his approach to facilitating Fermi problems, moving from direct instruction to encouraging learner reasoning.

Journal, Week 3 – Emerging facilitation strategies
"*Today I asked fewer leading questions. When one group estimated the number of bricks in the wall incorrectly, another group challenged them with reasoning about layers and rows. I simply listened and encouraged them to explain their thinking. The class actually corrected itself. I realized I don't need to give the answer. My job is to help them explain why their thinking makes sense."*
At this midpoint, reflection-in-action became evident; Mr Langa began to trust peer reasoning as a learning resource rather than an obstacle to order. Clear signs of adaptive experimentation began to appear. Mr Langa increasingly used probing questions during lessons - "*What assumption are you making?" or* "*How could you check if your estimate makes sense?"*—rather than direct correction.
In addition, classroom observations showed that Mr Langa began to value learners' reasoning, even when it was numerically inaccurate. This represented a pivotal shift in the domain of practice, supported by reflection-on-action that informed subsequent enactment. He also reinterpreted classroom "noise" as productive dialogue in interview 3, week 3: "*At first I thought the noise meant chaos. But now I hear them arguing about the math."*

Journal, Week 4 – Mr Langa realising the value of productive struggle
"*It used to worry me when they argued. Now I see that the arguments mean they are thinking. They even use everyday examples - like counting steps to estimate distance. The noise has changed; it is thinking noise."*
This marked a cognitive shift because he reframed classroom "noise" as evidence of engagement and reasoning, a key moment of conceptual reframing in the IMPG.

Journal, Week 5 – Mr Langa's reflection-on-action and planning

"*Reading back through my notes, I can see my own growth. I plan to include one estimation question each week, even after this project ends. It helps learners express ideas in their own words, not just fill in formulas.*"
Here, reflection-on-action is explicit, demonstrating Mr Langa's intention to sustain the practice beyond the research period.

Journal, Week 6 – Consolidation of new beliefs
"*I used to think open problems were too advanced for this context, but I see now that they make learners confident to reason. My job has shifted from explaining to listening and guiding.*"
This excerpt indicates that at the end of the intervention, Mr Langa articulated a transformed professional identity in his journal entry - moving from a transmitter of procedures to a facilitator of reasoning. Across the six weeks, the journal entry reflections illustrate a trajectory from controlled instruction toward adaptive facilitation. Mr Langa's journal entries revealed iterative cycles of experimentation, noticing, and adjustment - mirroring the dynamic links among the personal, domain, and consequence components of Clarke and Hollingsworth's IMPG. The reflections also demonstrate Schon's dual processes of reflection-in-action during lessons and reflection-on-action between lessons.

To capture reflection-in-action, short classroom transcripts and fieldnotes were analyzed alongside Mr Langa's reflective journal. These illustrate how shifts in professional thinking emerged during lesson facilitation rather than only in post-lesson reflection.

Classroom Observation, Transcript, Week 2
Learner A: *Sir, can we just guess the number first?*
Mr Langa: *You can, but tell us what you are guessing based on. What information are you using?*
Learner B: *The school block has four floors, maybe each floor has the same number of windows.*
Mr Langa: *Good — so how could we estimate without counting each one?*
Fieldnote (researcher annotation): *Mr Langa initially stepped back instead of quieting the class. He wrote learners' ideas on the board, allowing multiple strategies to surface before summarising patterns.*

At this moment, reflection-in-action was evident. In the IMPG, the Domain of Practice (new facilitation move: open questioning) influenced the Domain of Consequence (more learner reasoning), which, in turn, reshaped the Personal Domain (beliefs about control and participation). The quick in-situ decision not to intervene prematurely created a link between practice and personal growth.

In addition, field notes confirmed that during later sessions, Mr Langa paused deliberately during debates, allowing learners to critique one another's assumptions before intervening. His questioning pattern shifted from factual recall toward conceptual scaffolding—asking learners to compare reasoning across groups, a facilitation move aligned with literature on orchestrating modeling tasks (Albarracin & Ärlebäck, 2025). This stage illustrated Schon's reflection-in-action as Mr Langa responded to emerging classroom dynamics, experimenting with less directive approaches while maintaining overall lesson flow. In IMPG terms, feedback from the domain of consequence (observing learners' engagement and reasoning) triggered a change in his personal domain, reinforcing belief in collaborative reasoning as a legitimate form of mathematical learning.

Consolidated Pedagogical Insight and Professional Confidence
In week 4, Mr Langa used probing questions to extend learners' reasoning rather than confirm answers. The atmosphere was collaborative; several learners referenced estimation strategies from previous weeks. Next is the excerpt from the classroom observation.

Classroom Observation Transcript, Week 4

Learner C: *If we estimate the number of plastic bottles sold in the school, we can start from one class and multiply.*
Mr Langa: *Before you multiply, how could you check if your assumption is realistic?*
Learner D: *We can count bins — each has about ten bottles … there are six bins near the tuckshop.*
Mr Langa: Excellent — *now, what happens if some bins are emptied more often than others?*
The episode from week 4 classroom observation showed that reflection-in-action triggered movement from the External Domain (influence of prior PD workshop on questioning techniques) to the Domain of Practice (implementation in class). The immediate learner responses generated new consequences (peer justification and argumentation), consolidating belief changes observed later in his journal.

By the final two sessions, Mr Langa demonstrated consolidated pedagogical insight and increased confidence in facilitating open-ended problems. He articulated this growth in the Week 6 interview.

Interview Transcript, Week 6
Mr Langa:“ *I used to think learners had to be quiet to be learning. Now I see they learn through talking, even arguing. I feel more comfortable not having all the answers.*”
I noticed students compared their solutions without my asking. That tells me they value reasoning. I plan to include one estimation challenge each fortnight, just to keep the habit alive.
Fieldnote (researcher annotation): *During the post-lesson interview, Mr Langa referred to the* “*noise of learning” and identified it as evidence of cognitive engagement.*

This stage represents reflection-on-action, reinforcing links between the Domain of Consequence (perceived learner engagement) and the Personal Domain (professional beliefs about facilitation). According to IMPG, such recursive connections drive sustainable professional growth. The episode marked the consolidation phase, where experimentation becomes embodied practice.

Mr Langa’s reflective journal from the same week echoed a deeper epistemological shift: “*Mathematics can be about thinking, not just calculating. I see how estimation helps them reason.”*

Furthermore, classroom observation notes captured notable behavioral changes: he listened longer before responding, revoiced learner ideas to the class, and invited comparisons between different solution paths. These facilitation moves revealed an emerging adaptive expertise—the ability to apply professional knowledge flexibly in response to complex classroom realities (Hatano & Inagaki, 1986).

Furthermore, Mr Langa’s reflections showed evidence of meta-reflective awareness—thinking about his own reflection process. After reviewing transcripts and field notes with the researcher, he commented:

“*I can see the pattern now. Each week, I tried something new, then thought about it afterward. That helped me grow.”*

This recursive process exemplifies the interplay of reflection-in-action and reflection-on-action driving movement across IMPG domains. Mr. Langa’s final reflections revealed not only altered facilitation techniques but also reconfigured beliefs about mathematics, learners, and his professional identity. Although learners’ engagement and reasoning were evident in classroom discourse, this study does not claim measured gains in mathematical achievement.

Discussion

This study sought to understand how a mathematics teacher’s professional growth unfolded as he implemented Fermi problems in a disadvantaged South African classroom. The findings revealed a trajectory of reflective transformation characterized by three overlapping stages—initial uncertainty, emergent adaptive facilitation, and consolidated pedagogical insight. This growth was theorized through the interplay of reflection-in-action and reflection-on-action (Schon, 1983) across the four domains of the IMPG (Clarke & Hollingsworth, 2002).

Reflection as the Engine of Professional Change

The study sought to provide empirical support for the view that reflection functions as the mechanism driving teacher growth (Schon, 1983; Korthagen, 2017). During the intervention, Mr Langa's reflective processes were triggered by moments of cognitive dissonance - his expectations of control and correctness clashed with the unpredictable nature of learner reasoning. These disorienting moments acted as learning opportunities, compelling him to reframe problems of practice (Schon, 1983). This aligns with Chirinda's (2021) findings, which showed that reflective inquiry supports teachers in intentionally enhancing their professional growth.

Reflection-in-action enabled him to respond adaptively during lessons - pausing to listen, posing clarifying questions, or allowing learners to justify their reasoning. Reflection-on-action, conducted through post-lesson interviews and journals, deepened his understanding of what these moments signified for his teaching. Over time, these dual reflective processes became habitual, representing the internalization of reflective professionalism rather than an externally prompted behavior.

This aligns with Jaworski's (2005) notion of inquiry as a stance, where teachers continuously interpret and reinterpret practice rather than apply fixed techniques. The reflective cycles observed here thus represent not only momentary adjustments but also enduring epistemological change - from viewing teaching as transmission to seeing it as facilitation of meaning-making.

Mapping Growth Across Interconnected Domains

Clarke and Hollingsworth's (2002) IMPG provides a structural lens to trace how these reflections translated into sustained professional change. New pedagogical stimuli (the introduction of Fermi problems) activated the External Domain. Initial enactment revealed tensions in the Domain of Practice between traditional methods and open-ended demands.

As he experimented and observed unexpected learner engagement, new understandings emerged in the Domain of Consequence - Mr Langa began to perceive noise as discussion and diverse answers as evidence of reasoning. Reflection on these outcomes triggered transformation in his Personal Domain, shifting his beliefs about what counts as mathematical competence and effective teaching.

This non-linear interplay among domains exemplifies what Clarke and Hollingsworth (2002) describe as a change network rather than a progression. Reflection served as the crucial link between enactment and consequence, ensuring experiential insights were internalized and reapplied in subsequent lessons. This journey demonstrates that growth is not simply the accumulation of strategies but the reconstruction of professional meaning systems.

Fermi Problems as Catalysts for Situated Professional Learning

This study highlights the value of Fermi problems as catalysts for situated, practice-based professional learning. Unlike formal workshops that emphasize generalized pedagogy, Fermi problems situate teacher learning within authentic classroom activity. They compel teachers to navigate uncertainty, interpret learner thinking, and balance conceptual openness with curricular constraints - all of which generate the reflective tensions that promote growth.

For Mr. Langa, these tasks transformed everyday challenges—overcrowding, linguistic diversity, and varying learner abilities - into resources for professional inquiry rather than obstacles. This aligns with Cochran-Smith and Lytle's (2009) argument that teacher learning is most powerful when rooted in practice-as-inquiry. By using Fermi problems, the teacher not only developed facilitation techniques but also reconstructed his professional identity as a reflective and adaptive practitioner.

Moreover, the findings suggest that professional growth can occur without formal external inputs when teachers engage in guided reflection on their own practice. The cycles of reflection, enactment, and consequence observed in this study mirror the processes typically cultivated in structured PD programs (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011). However, in this case, growth emerged organically from authentic pedagogical work - underscoring the potential of Fermi problems as low-cost, sustainable tools for teacher learning in resource-constrained contexts.

From Procedural Certainty to Adaptive Expertise

A central insight from the study is the teacher's shift from procedural certainty - a reliance on fixed routines and correct answers - to adaptive expertise, characterized by flexibility, responsiveness, and conceptual focus (Hatano & Inagaki, 1986). This transformation was not immediate; it emerged through iterative negotiation between professional beliefs and classroom realities.

Initially, Mr. Langa equated control with effectiveness. Through repeated cycles of reflection and enactment, he reinterpreted uncertainty as an opportunity for professional inquiry. By the end of the intervention, he demonstrated hallmarks of adaptive expertise: openness to ambiguity, sensitivity to learner reasoning, and capacity for in-the-moment decision-making.

This finding reinforces the idea that professional growth involves not merely skill acquisition but a shift in epistemological orientation - from viewing mathematics teaching as the delivery of content to conceiving it as the orchestration of reasoning. Schon's framework helps explain this transformation as a movement from technical rationality to reflective practice, while Clarke and Hollingsworth's IMPG captures its systemic nature across domains.

Implications for Professional Development

Findings contribute to the pedagogical research discourse by illustrating how teachers' reflection cycles can be structured within lesson planning, using open-ended tasks as professional learning scaffolds. The teacher's reflective growth trajectory offered several implications for professional learning policy and practice. PD should be structured around authentic classroom experiences that invite teachers to confront uncertainty and reflect on real challenges. Fermi problems can serve as accessible entry points for teachers to explore mathematical reasoning and reframe their pedagogical identities. The post-lesson interviews and reflective journals in this study functioned as a mirror for professional thinking. School-based collaborative reflection sessions could replicate this mechanism sustainably. Teacher change should be understood as ongoing negotiation across domains of practice, belief, and consequence, mediated by reflection.

These implications emphasize that reflective professional growth can flourish even in under-resourced schools when teachers are supported to learn through practice. Overall, the discussion highlights that Fermi problems can operate simultaneously as an instructional innovation and a PD vehicle. By integrating Schon's process view of reflection with Clarke and Hollingsworth's structural model of growth, this study conceptualizes teacher learning as a self-sustaining system of reflection and enactment. The case of Mr. Langa illustrated how a teacher in a challenging context can evolve from procedural compliance to adaptive expertise through reflective inquiry in his own classroom practice.

Pedagogical Implications

The findings from this case study extend beyond one teacher's experience to offer insights for mathematics teacher education and school-based professional learning. They demonstrate that reflection on open-ended classroom tasks can serve, in this case, as a potentially sustainable, low-cost model of PD in resource-constrained contexts. Based on these insights, several pedagogical implications can be drawn for supporting teacher learning and reflective practice:

• Use Fermi problems as recurring reflective tasks to help teachers build confidence in dealing with uncertainty.
• Combine lesson facilitation with structured reflection prompts both during and after lessons.
• Encourage teachers to see classroom "noise" as a form of collaborative reasoning.
• Develop peer-reflection communities to maintain adaptive expertise in low-resource schools.

These implications highlight the practical potential of integrating Fermi problems and structured reflection into professional learning frameworks aimed at cultivating adaptive, inquiry-oriented teaching. For teacher educators, this case underscores the value of embedding reflection frameworks in pre-service methods courses to cultivate adaptive expertise early in professional formation. For school leaders, these findings highlight the importance of creating time and space for reflective dialogue after classroom experimentation. Future research examining sustained enactment and collective teacher reflection is needed to determine the durability and broader applicability of these practice-informed implications.

Conclusion

This study explored how a South African mathematics teacher's professional learning evolved through the implementation of Fermi problems in a resource-constrained classroom, using Schon's (1983) Reflective Practitioner and Clarke and Hollingsworth's (2002) IMPG as interpretive lenses. Findings revealed that iterative cycles of reflection-in-action and reflection-on-action enabled the teacher to shift from procedural certainty toward adaptive facilitation, reframing uncertainty as inquiry rather than control. This trajectory, mapped across the interconnected domains of practice, consequence, and personal growth, illustrates that teacher learning can be embedded within authentic classroom activity rather than dependent on external workshops. The study contributes to mathematics education by demonstrating how Fermi problems can function as both pedagogical tools and drivers for reflective PD, particularly in under-resourced schools. While this study does not provide evidence of long-term sustainability or scalability, it offers analytic insight into how reflective professional learning can emerge within everyday classroom practice under constrained conditions. To scale such reflective practice, professional learning should integrate structured reflection and open-ended tasks as low-cost, collaborative tools. Teacher education programs should explicitly teach reflective frameworks, while in-service development should prioritize inquiry-oriented lesson cycles that support contextual adaptation. Although limited to one case and context, the study offers analytic generalization to similar environments and highlights the potential for further research into collective reflection, long-term sustainability, and links between teacher reflection and learner outcomes. Overall, the study affirms that reflective, practice-embedded professional learning can empower teachers to develop adaptive expertise and sustain meaningful instructional innovation even under challenging conditions.

Declarations

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Availability of data and material: Any requests for data should be addressed to the author, who will share in accordance with what is allowed by the signed consent forms
Authors' contributions: B.C. is the sole author of the article

Ethics approval: Ethical approval for the study was obtained from the participating school and the university's ethics committee
Consent to participate: Participation in the study by the teacher was voluntary, and they could withdraw from the project at any time without negative consequences.
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Appendix A: Teacher Facilitation Guide for Fermi Problems

The teacher plays a crucial role in implementing Fermi problems effectively. This guide provides steps and tips for facilitating the sessions and ensuring maximum learner engagement.

1. Introduce the Problem Clearly: Present the Fermi problem orally and visually. Use real-world contexts that learners can relate to.
2. Encourage Group Brainstorming: Allow learners to discuss and identify assumptions before calculations begin.
3. Guide Without Giving Answers: Ask probing questions like 'What do you already know?' or 'How can you estimate that value?'
4. Support Mathematical Reasoning: Remind learners of relevant mathematical concepts (e.g., area, volume, rates).
5. Facilitate Whole-Class Discussion: After group work, bring the class together to compare solutions and highlight diverse thinking.
6. Encourage Reflection: Use the reflection instrument at the end of each session to reinforce metacognition.
7. Assess Process, Not Just Answers: Focus on reasoning, assumptions, and strategies rather than exact numbers.
8. Adapt for Large Classes: Assign clear group roles (e.g., recorder, presenter, checker) to manage participation.

Biographical note:

Brantina Chirinda is a mathematics education researcher and a teacher educator. She is a postdoctoral researcher at the University of California, Berkeley, and a senior research associate at the University of Johannesburg, South Africa. Dr Chirinda holds a PhD in Mathematics Education from the University of Witwatersrand, South Africa. Her primary research interest is in the teaching and learning of mathematics in underserved populations. Using Design-Based Research and sociocultural approaches, her research focuses on i) generating interventions that assist in providing students with meaningful opportunities to engage with rich mathematical content and practices, particularly mathematical thinking and problem solving, and ii) exploring and understanding ways to support teachers in underserved schools to make their classrooms more equitable.